Cournot Oligopoly

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April 24, 2020

Abstract

An *oligopoly* involves strategic interaction between two or more firms. This strategic interaction is modeled as a game. Unlike perfect competition, firms in an oligopoly do not take the market price as given and unlike monopoly, no single firm controls the entire market. Our beginning models of oligopoly will consider markets for homogeneous products, i.e., a market in which all firms produce an identical product. In other words, there is no *product differentiation*. We start with the **Cournot Oligopoly**. In this model, firms **simultaneously** choose their outputs. With the objective to maximize its profits, each firm chooses its output as a *best response* to the profile of outputs chosen by all the other firms. This gives the Nash equilibrium of the game. We will later see why this equilibrium outcome is not *pareto optimal*.

The General Model

A single good is produced by n firms. The cost to firm i of producing q_i units of the good is $C_i(q_i)$, where C_i is an increasing function, i.e., more output is more costly to produce. All the output is sold at a single price, which is determined by the demand for the good and the total output produced by the n firms.

The total output is given by Q $(= q_1 + q_2 + ... + q_n)$. The market price is given by $P(Q) = P(q_1 + q_2 + ... + q_n)$, where $P(.)$ is the inverse demand

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function. Note that firm i's revenue is given by $q_iP(q_1 + q_2 + ... + q_n)$. Hence, firm i's profit, is

$$\Pi_i = q_iP(q_1 + q_2 + ... + q_n) - C_i(q_i)$$

This industry is modeled as **Cournot's Oligopoly Game**:

The n firms are the *players*.

Each firm's set of *strategies* is the set of its possible outputs ($q_i \geq 0$).

Each firm's *preferences* are represented by its profit function Π_i.

Cournot Duopoly

We discuss a simple example with $n = 2$ firms (the industry is a *duopoly*). Let each firm i's cost function is given by $C_i(q_i) = cq_i$, where $c > 0$ is the common constant marginal cost for both firms (i.e., if the constant marginal costs of firms 1 and 2 are given by c_1 (> 0) and c_2 (> 0) respectively, then $c_1 = c_2 = c$)[1] and the inverse demand is given by

$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha. \end{cases}$$

where $\alpha > 0$. Assume $c < \alpha$.

If the firms' outputs are q_1 and q_2, then $Q = q_1 + q_2$. $P(q_1 + q_2)$ is α - q_1 - q_2 if $q_1 + q_2 \leq \alpha$ and 0 if $q_1 + q_2 > \alpha$. Thus firm 1's profits are

$$\Pi_1 = q_1(P(q_1 + q_2) - c) = \begin{cases} q_1(\alpha - q_1 - q_2 - c) & \text{if } q_1 + q_2 \leq \alpha \\ -cq_1 & \text{if } q_1 + q_2 > \alpha. \end{cases}$$

Note that we look for firm 1's best response to firm 2's output in the region where there's a possibility of firm 1's profit Π_1 being non-negative, i.e., in the region where $q_1 + q_2 \leq \alpha$. We will be looking to maximise the function $q_1(\alpha$ - q_1 - q_2 - $c)$ with respect to q_1 for given q_2 to get the best response function $q_1(q_2)$.

Now, carefully look at the expression $q_1(\alpha$ - q_1 - q_2 - $c)$(This is the profit when $q_1 + q_2 \leq \alpha$).

For any $q_2 > \alpha$ - c, any positive $q_1(q_2)$ will give negative profits. Therefore, it is best response to choose $q_1(q_2) = 0$ for $q_2 > \alpha$ - c. The positive best

[1] There is a homework assignment at the end of this section with $c_1 \neq c_2$.

response $q_1(q_2)$ for $q_2 \leq \alpha$ - c is obtained by partially differentiating $q_1(\alpha$ - q_1 - q_2 - $c)$ with respect to q_1 and equating the partial derivative to zero.

$$\frac{\partial[q_1(\alpha - q_1 - q_2 - c)]}{\partial q_1} = 0 \tag{1}$$

implies

$$q_1(q_2) = \frac{\alpha - c - q_2}{2} \tag{2}$$

More formally,

$$q_1(q_2) = \begin{cases} (\alpha - c - q_2)/2 & \text{if } q_2 \leq \alpha - c \\ 0 & \text{if } q_2 > \alpha - c. \end{cases} \tag{3}$$

Similarly we can show that

$$q_2(q_1) = \begin{cases} (\alpha - c - q_1)/2 & \text{if } q_1 \leq \alpha - c \\ 0 & \text{if } q_1 > \alpha - c \end{cases} \tag{4}$$

Solving equations (3) and (4) simultaneously[2] we get the Nash equilibrium

$$(q_1^*, q_2^*) = (\frac{\alpha - c}{3}, \frac{\alpha - c}{3})$$

Cournot's game with n firms

Firm 1's payoff is

$$\Pi_1 = \begin{cases} q_1(\alpha - q_1 - q_2 - ... - q_n - c) & \text{if } q_1 + q_2 + ... + q_n \leq \alpha \\ -cq_1 & \text{if } q_1 + q_2 + ... + q_n > \alpha \end{cases}$$

Firm 1's best response function is

$$q_1(q_{-1}) = \begin{cases} (\alpha - c - q_2 - .... - q_n)/2 & \text{if } q_2 + ... + q_n \leq \alpha - c \\ 0 & \text{if } q_2 + ... + q_n > \alpha - c \end{cases}$$

The best response function of every other firm is the same. In an equiibrium (q_1^*, q_2^*, ..., q_n^*) in which all firms' outputs are positive,

[2] Draw the best response curves and note their intersection. You may consult the readings.

$$q_1^* = \frac{\alpha - c - q_2^* - q_3^* - ... - q_n^*}{2}$$

$$q_2^* = \frac{\alpha - c - q_1^* - q_3^* - ... - q_n^*}{2}$$

. . . . . .

$$q_n^* = \frac{\alpha - c - q_1^* - q_2^* - ... - q_{n-1}^*}{2}$$

We can write these equations as

$$0 = \alpha - c - 2q_1^* - q_2^* - ... - q_n^*$$

$$0 = \alpha - c - q_1^* - 2q_2^* - ... - q_n^*$$

. . . .

$$0 = \alpha - c - q_1^* - q_2^* - ... - 2q_n^*$$

If we subtract the second equation from first we obtain $q_1^* = q_2^*$. If we subtract the third equation from the second we obtain $q_2^* = q_3^*$, and so on. We will get $q_1^* = q_2^* = q_3^* = ...= q_n^* = q^*$. Put this common value q^* into any one of the n equations above and we get $q^* = (\alpha - c)/(n + 1)$. That is, the game has a unique Nash equilibrium where the output of each firm i is $(\alpha - c)/(n + 1)$.

This is a general result for n firms. Put $n = 2$ you get the Nash equilibrium of the Cournot duopoly, i.e., the output of each of the two firms is $(\alpha - c)/3$.

Essentially, the general version of the Cournot game is as follows:

Firm i's profit is,

$$\Pi_i = q_i P(Q) - C_i(q_i)$$

where

$$Q = q_1 + q_2 + ... + q_n$$

We find each firm i's best response by taking the first order condition of the objective function with respect to q_i:

$$\frac{\partial \Pi_i}{\partial q_i} = P(Q) + q_i \frac{\partial P(Q)}{\partial q_i} + \frac{\partial C_i(q_i)}{\partial q_i} = 0$$

The equation must hold for all i = 1,2, ..., n in the Nash equilibrium.

Homework Assignment (This assignment is graded. The solutions to the following problems should be submitted to econ14vibhor@gmail.com on April 30, 2020 10 pm IST.)

Problem 1. Draw the best response curves of both firms by carefully following the best response functions in the Cournot duopoly example discussed above. Show the Nash equilibrium. **(10 points)**

Problem 2. Consider that the constant marginal costs of firms 1 and 2 are c_1 and c_2 respectively where $c_1 \neq c_2$. Find the Nash equilibrium where $c_2 > (\alpha + c_1)/2$ (Hint: Write the best response functions and draw the best response curves). **(10 points)**